is quite obvious that the air required for ventilation would not only have to be supplied in huge quantities by special fans, but also that it would have to be carried away in closed ducts to the outside of the engine-room.

When dealing with gas engines for alternating-current generators the author falls into a strange error. He mentions as one of the drawbacks that power is lost through the damping coils which the irregular motion of a gas engine renders necessary. Now it is well known that damping coils must not be used in such cases. His remarks on slot insulation, on which subject he is an authority, are highly interesting; he believes that eventually it will be possible to reduce this to something like 2 mm. for a 10,000-volt machine, but unfortunately he does not say in what manner this improvement is to be achieved. He is evidently an advocate of severe testing, and the subject of insulation tests gives him the opportunity of a homily on the ethics of the inspecting engineer. His suggestion that the sufficiency of the mechanical support of the winding should be tested by short-circuiting at full excitation the terminals of, say, a 5000-kw. alternator sounds rather heroic, and his anticipation that not more than six times normal current would flow at the instant of closing the switch may be doubted, although he is quite right in saying that, a moment after, the current would only be about three times the normal value.

The chapter on the design of the central station as a whole is particularly interesting and useful. Here we find an enormous mass of information collected from a variety of stations and tabulated in a convenient form. The same may be said of the chapter on transmission plant. The author gives us, not only technical details, but also the cost from actual experience, and one cannot but admire the industry with which he has collected so much really valuable information. As regards electric traction, his sympathies are all for the direct-current system, for which he predicts a rise of working pressure up to something like 1200 volts. The single-phase system he condemns entirely, but as regards the three-phase he admits a slight superiority in the matter of weight over the direct-current system. As the limits of the power of motors at the ordinary one-hour rating he takes 150 h.p. for the single-phase, 300 h.p. for the continuous, and 400 h.p. for the three-phase system. The three-phase system is a little lighter, and the single-phase system more than twice as heavy as the continuous-current system. A new and very simple formula for the tractive resistance in kg. per ton of train is given on p. 231. It is as follows:—

$$R = 2{\cdot}70 + 0{\cdot}09 \frac{V^2}{W}$$

for railways in the open, and

$$R = 3 + 0{\cdot}3 \frac{V^2}{W}$$

for tube railways. V is the speed in km. per hour, and W is the weight of the train in tons.

GISBERT KAPP.

WHY LEAVES ARE GREEN.

Zur Biologie des Chlorophylls, Laubfarbe und Himmelslicht, Vergilbung und Etiolement. By Ernst *Stahl.* Pp. v+153. (Jena: Gustav Fischer, 1909.) Price 4 marks.

IN this interesting and suggestive book, Prof. Stahl presents us with the results of his observations and speculations upon the ever-interesting problems of the biology of chlorophyll and its related colouring matters. One of the most interesting of these is the cause of the prevailing green colour of our vegetation. How does it arise that the various photosynthetic organs of plants are green, and not some other colour?

Engelmann has already shown that the colours of the algal vegetation of the sea are complementary to the light which falls upon them, and Gaidukov has made experiments to show that the Cyanophyceæ, or blue-green algæ, undergo a change in colour complementary to the light which falls upon them, when grown under different coloured lights. Prof. Stahl thinks that these observations may lead to an explanation of the green colour of land plants. The chlorophyll spectrum may be regarded as a combination of two absorption spectra. The absorption at the blue end of the spectrum agrees very nearly with that of etiolin and the colouring matter of yellow leaves, whilst the absorption in the red corresponds to that of the green colouring matter which is formed when etiolated plants are exposed to light, and disappears in the autumn, when the leaves again turn yellow. The yellow-green colour of the leaf may, therefore, be an adaptation to the prevailing colour of the diffuse light which falls upon it, the yellow being complementary to the blue of the heavens, and the green to the orange and red which mostly prevail when the sun is low.

The region of least absorption in the chlorophyll corresponds with that of maximum energy in the spectrum. The plant does not, therefore, depend for its assimilative work upon the rays of greatest energy. On the other hand, the possibility of using these rays is shown by the red algæ, which absorb the green as well as the blue, the maximum of their assimilative activity lying exactly in the green.

The author tries to show that the non-absorption of the green rays is not only due to the fact that the chlorophyll makes no use of those rays which usually reach it in a weakened form, but also to the fact that the absorption of these rays in direct sunlight would be dangerous to the plant, because of their great heating power. Under normal conditions an intense illumination is unnecessary. The amount of energy used up by the chlorophyll grain in carbohydrate assimilation is only a small part of the total energy it absorbs. In light of lower intensities, however, it is clear that the amount of energy absorbed by the plant becomes more nearly proportional to the amount used for assimilation, and thus a complementary colour adaptation to the light is understandable. In the red and brown seaweeds, the blue-green algæ, &c., the absorption of

the green rays may be necessary to supply the plant with the energy required. A too intense illumination of the leaf, by concentration of the sun's rays upon it, destroys the chlorophyll. According to Pringsheim, this is caused by the chemical rays, but Prof. Stahl considers that the effects of the heat rays have been overlooked, and he insists on this as an important factor in the problem. The variation in the colour of foliage leaves, according to whether they are in the sun or in the shade, is partly due, he thinks, to this danger of overheating. In the special case of the red and brown seaweeds, he considers that the colours are not entirely due to an adaptation to the quality of the light, but also to its intensity.

How far the author's conclusions are justified remains to be seen, but he adduces a considerable amount of evidence in favour of them, which he discusses in a most interesting and suggestive way.

Prof. Stahl suggests that the etiolation, and the yellow coloration of leaves in autumn, may be due to the need of economy in food materials. Willstätter has shown that, in its purest form, green chlorophyll contains C, H, O, N, and Mg. The yellow colouring matters contain only C, H, and O, so that, by keeping back the green chlorophyll in the spring and re-absorbing it in the autumn, a saving would be effected in nitrogen and magnesium, which are of great value to the plant.

Some interesting experiments are described to show that this actually does take place. If leaves which are just on the point of turning yellow, but are still green, are removed from the plant and kept in a damp chamber, they retain their green colour, whilst neighbouring leaves, still attached to the plant, become yellow. So, also, if slits are cut in the leaf, so that the principal veins are severed, the portions of leaf thus cut off from the main conducting vessels remain green, whilst the other parts turn yellow. The results of experiments made by various observers, and others recently made at the author's suggestion, in the agricultural laboratory at Jena, are brought forward to show that potassium and nitrogen, phosphoric acid, iron, chlorine, and silica, are more or less reduced in amount in the yellow as compared with the green leaf. The significance of these facts, which no doubt lend considerable support to Prof. Stahl's interesting hypothesis, is fully discussed, but that the etiolation of young leaves and the yellow coloration of old leaves are so definitely associated with the plant's need for economy cannot, from the evidence before us, be said to be so clearly established as Prof. Stahl seems to think.

H. W.

THE FOUNDATIONS OF GEOMETRY.

Grundlagen der Geometrie. By D. Hilbert. Third edition. Pp. vi+280. (Leipzig and Berlin: B. G. Teubner, 1909.) Price 6 marks.

THIS fascinating work has long since attained the rank of a classic, but attention may be directed to this new edition, which has various additions, mainly bibliographical, and seven supplements, which are reprints of papers by the author on topics related to that of his famous essay. Two of these can be enjoyed by readers with no exceptional mathematical knowledge. In the one on the equality of the base angles of an isosceles triangle, Dr. Hilbert proves, *inter alia,* the remarkable fact that, even if we assume Euclid's theory of proportion, we cannot prove his propositions on equalities of area, unless we assume the truth of prop. 4, bk. i., of the "Elements" in the wider sense—that is, when one triangle has to be turned over to make it fit the other. It is also pointed out (p. 68) that two tetrahedra can be constructed with equal heights, and bases of equal area, which cannot be cut up into congruent polyhedra, and to which congruent polyhedra cannot be added in such a way that the solids thus produced can be sliced up into congruent parts. Consequently it is impossible to build up a theory of equality of volumes strictly analogous to Euclid's theory of equality of areas.

Another supplement of general interest, and easily understood, is that on the notion of number. The most noticeable thing here is the remark that the commutative law of addition $(a+b=b+a)$ can be deduced from the distributive laws of multiplication, together with the axiom $a.1=1.a=a$; thus

$$(a+b)(1+1)=(a+b).1+(a+b)1=a+b+a+b$$
$$(a+b)(1+1)=a(1+1)+b(1+1)=a+a+b+b;$$

therefore $a+b+a+b=a+a+b+b$, and hence $b+a=a+b$.

The seventh supplement, on the foundations of logic and arithmetic, deserves very careful study, both by mathematicians and by philosophers. The main feature of this is that an aggregate is defined as *any* object of thought, and the notion of "element of an aggregate" is a derived one. Dr. Hilbert objects to Dedekind's method in his well-known tract on number, because it postulates the aggregate of "all objects of thought" as a definite conception. A sort of promise is given that the author will expand the ideas of this essay in greater detail, and it is earnestly to be hoped that this intention will be carried out. In connection with these discussions there is one point that deserves attention; a finite intelligence thinks *in time,* and cannot rid itself of that idea. Now, if we take the statements (1) I am conscious; (2) I am conscious that I am conscious; (3) I am conscious that I am conscious that I am conscious; (1) is the most elementary possible thought from a metaphysical point of view, (2) is the most elementary form of reflection, and if we admit that any thought can be reflected upon, we at once get the natural scale in the form t, tr, tr^2, tr^3, &c. It is not impossible that some such reasoning was in the mind of Rowan Hamilton when he made the statement, which puzzled De Morgan, that "Algebra is the science of Pure Time." Until time is defined in terms of simpler entities, it is open to question whether any generation of the natural scale is really more fundamental than the above. Of course, there may be methods which are preferable in the eyes of a mathematician who wishes to avoid metaphysical discussion; but the fact remains that there is a metaphysical aspect of the question which must be faced before a final answer is reached.

G. B. M.